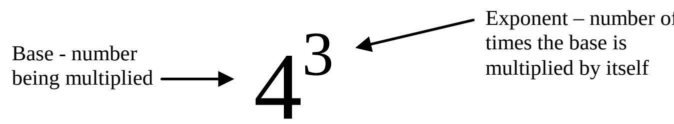
Exponents

Introduction

Whole number exponents stand for repeated multiplication.

For example, 4^3 or "four to the third power," means 4 times *itself* 3 times, or 4 x 4 x 4, or $4 \cdot 4 \cdot 4$
The example, 4^3, equals 64 because $4 \cdot 4 = 16$, which, multiplied by 4 a third time, equals 64.

In the example above, 4 is called the **base**, and 3 is called the **exponent**.



Another example:

$8^4 = 8 \cdot 8 \cdot 8 \cdot 8$

$8 \cdot 8 = 64$
$64 \cdot 8 = 512$
$512 \cdot 8 = 4096$
So, $8^4 = 4096$

In general, b^n is b times itself n times.

The term "squared" means raised to the second power. Three squared is 3^2 or $3 \cdot 3$
The term "cubed" means raised to the third power. Four cubed is 4^3 or $4 \cdot 4 \cdot 4$

Special Exponents

Any number to the power of 1 is the number you started with. For example:

$9^1 = 9$ $\quad 4^1 = 4$ $\quad 1^1 = 1$ $\quad 1000^1 = 1{,}000$ $\quad b^1 = b$

Any number to the 0 power is 1. For example:

$9^0 = 1$ $\quad 4^0 = 1$ $\quad 1^0 = 1$ $\quad 1000^0 = 1$ $\quad b^0 = 1$

Multiplying Exponential Expressions

When multiplying exponential expressions with the same base, keep the base the same and add the exponents. For example:

$4^3 \cdot 4^5 = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = 4^8$ $\quad 7^2 \cdot 7^3 = (7 \cdot 7) \cdot (7 \cdot 7 \cdot 7) = 7^5$ $\quad b^m \cdot b^n = b^{m+n}$

Dividing Exponential Expressions

When dividing exponential expressions with the same base, keep the base the same and subtract the exponents. For example:

$3^5 \div 3^2 = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3^3$ $\quad 7^4 \div 7^3 = \frac{7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7} = 7^1 = 7$ $\quad \frac{5^4}{5^2} = \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^2$ $\quad \frac{b^m}{b^n} = b^{m-n}$

Negative Exponents

A base with a negative exponent is its reciprocal (fraction turned upside down) with a positive exponent. A negative exponent signals you to turn the expression into a fraction and put the term with the negative exponent on the opposite level of the fraction as a positive exponent.

For example:

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25} \qquad 6^{-3} = \frac{1}{6^3} \qquad \frac{1}{9^{-4}} = 9^4 \qquad b^{-y} = \frac{1}{b^y}$$

$$\frac{1}{3^{-4}} = 3^4 \qquad \frac{1}{5^{-3}} = 5^3 \qquad \frac{3^{-2}}{5^{-4}} = \frac{5^4}{3^2} \qquad \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 \qquad \frac{1}{b^{-y}} = b^y$$

Exponents Raised to a Power

Exponential expressions can be raised to a power themselves. When raising an exponential expression to a power, keep the base the same and multiply the exponents.

For example:

$(8^3)^2$ is $8^3 \cdot 8^3$ or $(8 \cdot 8 \cdot 8) \cdot (8 \cdot 8 \cdot 8)$ or 8^6 $\qquad (5^2)^5 = 5^{10} \qquad (2^4)^9 = 2^{36} \qquad (b^m)^n = b^{m \cdot n}$

Exponents and Fractions

When a fraction is raised to a power, raise the numerator and denominator to the exponent.

$$\left(\frac{1}{3}\right)^2 = \frac{1^2}{3^2} = \frac{1}{9} \qquad \left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125} \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Fractions in Exponents

When an exponent is a fraction, use the denominator of the exponent to tell you what root to take of the base. Then raise that to the power of the numerator.

For example:

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9 \qquad 36^{\frac{1}{2}} = (\sqrt[2]{36})^1$ (also written $\sqrt{36}$) $= 6$

$25^{\frac{3}{2}} = (\sqrt[2]{25})^3 = 5^3 = 125 \qquad x^{\frac{a}{b}} = (\sqrt[b]{x})^a$

Try These

$4^{[illegible]}$		$\frac{10^7}{10^2}$		$(9^2)^4$	
$62^{[illegible]}$		12^{-2}		$\left(\frac{1}{2}\right)^3$	
$3^5 \cdot 3^7$		$\frac{1}{6^{-3}}$		$81^{\frac{1}{2}}$	
$9^6 \div 9^3$		$\left(\frac{3}{5}\right)^{-5}$		$16^{\frac{3}{2}}$	