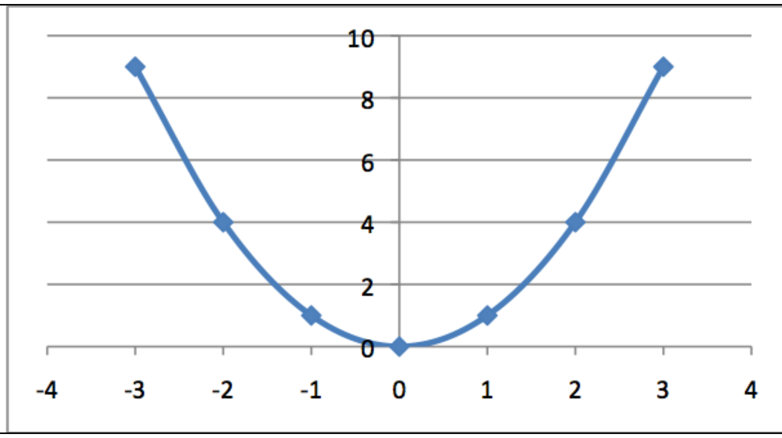
Steps to Transform a Simple Parabolic Graph

Given the equation $y=-\frac{1}{4}(x-2)^2+4$, how can you figure out what the graph looks like without calculating and plotting a bunch of points?

Step 1

First, determine what simple graph this would be similar to. Since we know we are working with a parabolic graph, it looks like $y=x^2$. (Why? Because the x term – the part within the parentheses – is squared.) If you don't know what this graph looks like, you should graph it.

x	**$y=x^2$**
3	9
2	4
1	1
0	0
-1	1
-2	4
-3	9



As you work through this exercise, keep an eye on the vertex of this parabola (the point where the two arms meet). The parabola may change shape as we expose different parts of it, but watching where the vertex moves will help to keep you oriented.

Step 2

Identify the ways in which the equation you need to graph is different from the simple graph. Compare:

$$y=x^2 \quad \text{vs.} \quad y=-\frac{1}{4}(x-2)^2+4$$

The parts highlighted in red are different than the original equation. There appear to be three main differences, but there are actually four things we need to be concerned with. To make this clear, let's re-write the equation:

$$y=-1\left(\frac{1}{4}\right)(x-2)^2+4$$

You probably recall from earlier math classes that a negative sign before a number is the same as multiplying that number by -1. When we break the equation out this way, we can see that there are four changes from the original equation that we should be concerned with. By taking the graph of $y=x^2$ and applying each of these changes in the correct order, we will arrive at the graph of $y=-\frac{1}{4}(x-2)^2+4$. We are going to work through this step by step, but once you have done the process a few times, you will probably be able to make the transformations in your head.

Step 3

Shift the graph horizontally

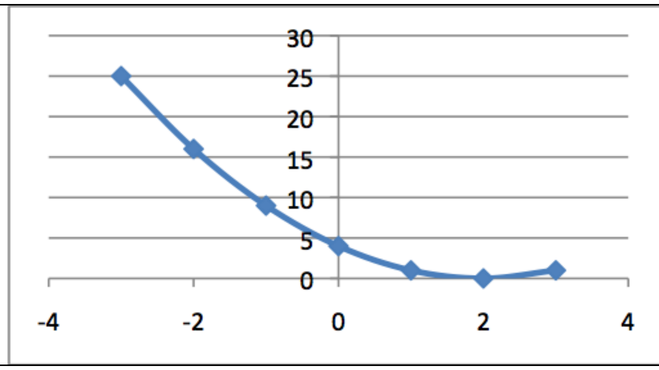
$$y=-1\ (1/4)\ (x-2)^2+4$$

A number added to or subtracted from the x term shifts the graph left or right. So, to tackle this step, we are going to make the change highlighted above to our graph of $y=x^2$. This gives us:

$$y=(x-2)^2$$

How we got here: $y=x^2 \rightarrow y=(x-2)^2$

x	$y=(x-2)^2$
3	1
2	0
1	1
0	4
-1	9
-2	16
-3	25



The first thing you probably noticed is that we lost most of one half of the graph. While the most apparent, this is actually the least meaningful change. We lost part of the graph because the effect of subtracting two from x was to shift the whole graph two units to the right. When we did that, the right arm of the parabola got chopped off by the margin of the graph. We could fix this by including x values greater than three in our graph. (If you have the time, try calculating the y values for x=4 and x=5 and see how adding them fixes the graph.) For this example, we're not going to fix the graph – we're going to keep using the same set of numbers all the way through – just keep in mind that it is still a parabola; we have just chopped off part of it.

Step 4

Compress or stretch the graph

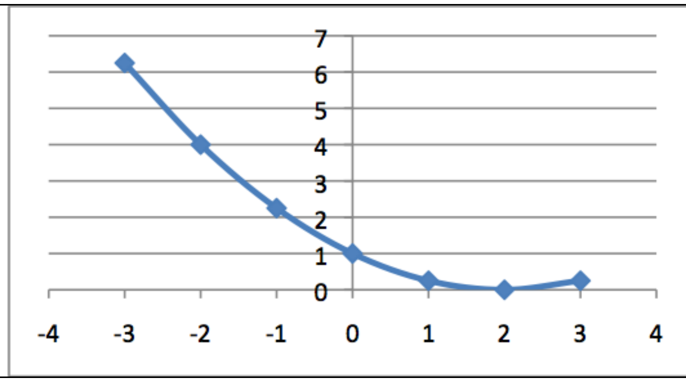
$$y=-1\ (1/4)\ (x-2)^2+4$$

The number preceding the x term is multiplied by the x term, so, if it is less than one (but greater than zero), it will compress the graph vertically and if it is greater than one, it will stretch the graph vertically. (If the number is negative, you forgot to transform that negative to a -1 as suggested in Step 2, so you should go back and fix that.) Adding the ¼ into the equation we used in Step 3 gives us:

$$y=\frac{1}{4}(x-2)^2$$

How we got here: $y=x^2 \rightarrow y=(x-2)^2 \rightarrow y=\frac{1}{4}(x-2)^2$

x	$y=\frac{1}{4}(x-2)^2$
3	¼
2	0
1	¼
0	1
-1	2¼
-2	4
-3	6¼



Notice that the graph from Step 3 passed through (-3,25) but now passes through (-3,6¼). 6¼ is one quarter of 25. The graph has been compressed to one quarter of its previous vertical span.

Step 5

Reflect the graph (change values from negative to positive)

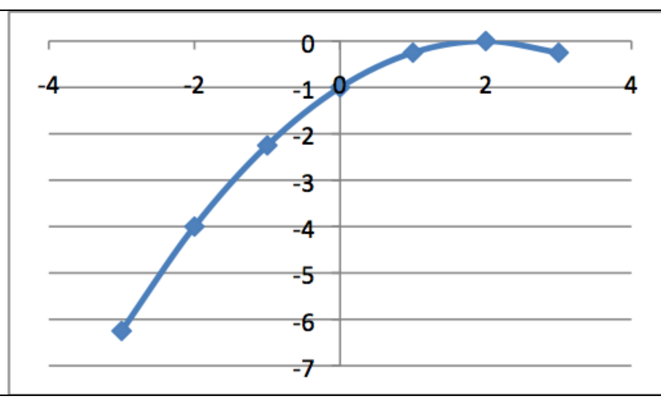
$$y=-1\,(\tfrac{1}{4})\,(x-2)^2+4$$

If there is a negative sign leading the equation, this will change the x value from positive to negative, which turns the graph upside down, reflecting it around the x axis.

$$y= -1(\tfrac{1}{4})(x-2)^2$$

How we got here: $y=x^2 \rightarrow y=(x-2)^2 \rightarrow y=\frac{1}{4}(x-2)^2 \rightarrow y=-1(\frac{1}{4})(x-2)^2$

x	$y=-1(\frac{1}{4})(x-2)^2$
3	-¼
2	-0
1	-¼
0	-1
-1	-2¼
-2	-4
-3	-6¼



Notice that where the graph from Step 4 passed through (0,1) and (-2,4), it now passes through (0,-1) and (-2,-4). We have reversed the y values when the graph was reflected across the x axis. The parabola previously opened upward and it now opens downward.

Step 6

Shift the graph up or down

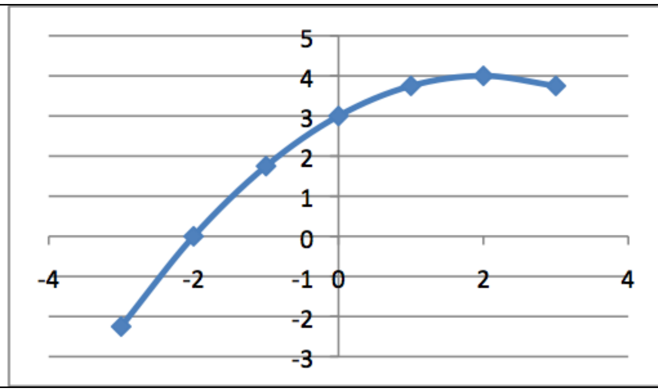
$$y=-1\ (1/4)\ (x-2)^2+4$$

A number added after the x value (but outside the parentheses) either shifts the graph upward (if positive) or downward (if negative).

$$y= -1(1/4)(x-2)^2+4$$

How we got here: $y=x^2 \rightarrow y=(x-2)^2 \rightarrow y= 1/4\ (x-2)^2 \rightarrow y= -1(1/4)(x-2)^2 \rightarrow y= -1(1/4)(x-2)^2+4$

x	$y=-1(1/4)(x-2)^2+4$
3	3¾
2	4
1	3¾
0	-1
-1	1¾
-2	0
-3	-2¼



Notice that the point (2,0) was an x-intercept in Step 5 but, after shifting up, it is now at (2,4), no longer touching the x axis. This is a change of +4, just as the equation indicates.

In this example, we started with the equation $y=x^2$. We then shifted the graph horizontally to the right by two, compressed the graph vertically by ¼, reflected the graph across the x axis, and finally shifted the graph upward by 4. The shape is still what we started with (a parabola), but the positioning and orientation of that shape have changed.

The graph produced in Step 6 is the graph of $y=-1/4\ (x-2)^2+4$, which is what you set out to solve. Of course, you could have skipped straight to this step, calculated points, and plotted the graph, but the purpose of this exercise is to learn how to transform graphs without having to explicitly plot points. If you practice these steps and make an effort to understand why each step works the way it does, you will develop the ability to transform graphs quickly in your head, which can be a very useful skill in higher math and science classes.